Nonlinear compression of solitary waves in asymmetric twin-core fibers

T. Soloman Raju,[1] Prasanta K. Panigrahi,[2,*] and K. Porsezian[1]
[1]*Department of Physics, Pondicherry University, Kalapet, Pondicherry, 605 014, India*
[2]*Physical Research Laboratory, Navrangpura, Ahmedabad, 380 009, India*
(Received 16 August 2004; published 22 February 2005)

We demonstrate a different pulse compression technique based on exact solutions to the nonlinear Schrödinger-type equation interacting with a source, variable dispersion, variable Kerr nonlinearity, and variable gain or loss. We show that this model is appropriate for the pulse propagation in asymmetric twin-core fibers. The chirped pulses are compressed due to the nonlinearity as well as dispersion management as also due to the space dependence of the gain coefficient. We also obtain singular solitary wave solutions, pertaining to extreme increase of the amplitude due to self-focusing.

DOI: 10.1103/PhysRevE.71.026608 PACS number(s): 42.81.Dp, 47.20.Ky

In recent years, the study of nonlinear fiber optics has attracted much attention and has played an important role toward the development of several technologies [1]. Among them, the development of optical solitons is considered to be one of the ten hottest technologies of the 21st century [2]. In the case of exact soliton pulse propagation, the pulse evolution is governed by nonlinear Schrödinger equation (NLSE). In realistic systems this equation is suitably modified to take into account loss or gain or other medium effects. In recent times, much effort has been devoted to optical pulse compression techniques because of their practical utility. Most of these techniques rely on chirping obtained either by self-phase modulation in the normal dispersion regime or by combining phase modulation with amplification [3,4]. Soliton effects can also be utilized for compression where the problem of residual pedestals can be reduced through appropriate control of intensity, which affects the nonlinearity. However, this procedure has the drawback of waste of energy [5]. Adiabatic soliton compression, through the decrease of dispersion along the length of the fiber, provides a better pulse quality [6], albeit in a less rapid manner. Interested readers are referred to Johnson *et al.* [7] and Fisher *et al.* [8] for more information about pulse compressors. Exact solutions have played crucial roles in demonstrating the above pulse compression techniques. The fact that NLSE or modifications of the same is known to possess soliton solutions has come in handy in studying the mechanism of pulse compression in the above models. All the aforementioned methods for pulse compression are restricted to pulse propagation through single core fibers. Although it is easier to fabricate twin-core fibers with some built-in asymmetry, the nonlinear pulse compression in these types of couplers has not received much attention in the literature. The existence of the solitary wave solutions in twin-core fibers (TCFs) has been reported in Refs. [9,10]. Soliton solutions, when the nonlinearity for one component can be neglected, has been studied perturbatively [11]. In this context, the relevant equation is NLSE driven by a source, originating from the coupling term. Soliton bound states in the TCFs have also been reported [12].

*Electronic address: prasanta@prl.ernet.in

In this paper we delineate the nonlinear pulse compression based on exact solitary wave solutions of NLSE interacting with a source, that is appropriate for the pulse propagation in asymmetric TCF. Apart from using the exact solutions of NLSE with a source, recently obtained by two of the present authors [13], we take recourse to the recent work of Kruglov *et al.* [14] in the context of NLSE with variable dispersion, variable Kerr nonlinearity, and variable gain or loss.

We first outline below the origin of NLSE with a source, for pulse propagation through asymmetric TCF, with dissipation [11]. The equations for the envelopes of the pulses that propagate through the TCF are

$$i\partial_z\psi_1 + \partial_{\tau\tau}\psi_1 + 2|\psi_1|^2\psi_1 + ig\psi_1 + \Gamma\alpha_{12}\psi_2 \times\exp[-i(\hat{k}z - \hat{\omega}\tau)] = 0, \tag{1}$$

$$i(\partial_z\psi_2 - \beta_1\partial_\tau\psi_2) + \beta_2\partial_{\tau\tau}\psi_2 + 2|\psi_2|^2\psi_2 + \frac{\alpha_{21}}{\Gamma}\psi_1\exp[i(\hat{k}z - \hat{\omega}\tau)] = 0. \tag{2}$$

Here ψ_1 and ψ_2 are the field envelopes. The coordinates z and τ in Eqs. (1) and (2) are written in appropriate units [15]. In writing Eqs. (1) and (2), we have considered constancy of the distributed coefficients. In any real soliton transmission system there exists dissipation due to fiber losses. This has been incorporated in Eq. (1), by adding an $ig\psi_1$ term. As the second core is a passive one, it is not essential to consider the losses. Since the fibers are not identical, the coupling is not symmetric, i.e., $\alpha_{12}\neq\alpha_{21}$. $\Gamma=(\gamma_1/\gamma_2)^{1/2}$ is the ratio of the nonlinearity strengths in the two fibers, where [16,17]

$$\gamma_i = \frac{n_2\omega_i}{cA_i^{\mathrm{eff}}}. \tag{3}$$

A_i^{eff} is the effective core area, n_2 is the Kerr coefficient, c is the speed of light, and ω_i is the carrier frequency in each fiber. Under the assumption that the interaction term in Eq. (1) is much larger than the interaction term in Eq. (2), the last term in Eq. (2) can be dropped. This implies that Eq. (2) is decoupled from Eq. (1); ψ_2 only enters as a driving term in Eq. (1), while there is no back action. We further assume that the pulses described by Eq. (2) are in the normal dispersion regime, in which case there is no modulational instability and stable linear dispersive waves can propagate in the second core. We are interested in the small amplitude modes of Eq. (2), when the pulses are just linear waves. In this case the term arising from the Kerr nonlinearity can be dropped. Thus Eqs. (1) and (2) can be written as damped NLSE coupled to an external traveling wave field.

In the realistic situation in a fiber, there will always be some nonuniformity due to two factors. It may arise from a variation in the lattice parameters of the fiber medium, so that the distance between two neighboring atoms is not constant throughout the fiber. It may also arise due to the variation of the fiber geometry, e.g., diameter fluctuation. These nonuniformities influence various effects such as loss (or gain), phase modulation, etc. These effects can be modeled by making dispersion, gain, and other space dependent parameters. In this case, Eq. (1) modifies to

$$i\psi_z - \frac{\beta(z)}{2}\psi_{\tau\tau} + \gamma(z)|\psi|^2\psi = i\frac{g(z)}{2}\psi + \eta(z)e^{i\Phi(\tau,z)}. \quad (4)$$

The above equation is deliberately cast into a form similar to that of Ref. [14], where the solutions of this equation without a source have been recently analyzed. The phase Φ in the source term contains the phase part of ψ_2 whose amplitude part is contained in η. Equation (4) describes the amplification or attenuation [for negative $g(z)$] of pulses propagating in a single mode nonlinear fiber, where $\psi(\tau,z)$ is the complex envelope of the electric field in a co-moving frame. τ is the retarded time, $\beta(z)$ is the group velocity dispersion (GVD) parameter, $\gamma(z)$ is the nonlinearity parameter, and $g(z)$ is the distributed gain function.

In recent times, various forms of inhomogeneities have been discussed in the literature. A nonlinear compression of chirped solitary waves has been discussed by Moores [18] and Shivkumar [19]. A deformed NLSE has been studied by Brustev *et al.* in Ref. [20], wherein the Lax pair for the system has been presented. The soliton solution and the possibility of amplification of soliton pulses using a rapidly increasing distributed amplification with scale lengths comparable to the characteristic dispersion length has been reported by Quiroga-Teixeiro *et al.* [21]. For the propagation of two orthogonally polarized optical fields in a nonuniform fiber media, the coupled inhomogeneous NLSE, under suitable variable transformation, has been reduced to the coupled NLSE [22]. Similarity reduction for variable-coefficient coupled NLSE has been studied in Ref. [23]. Numerically it was shown that, in the case where the gain due to the nonlinearity and the linear dispersion balance each other, equilibrium solitons are formed [24]. As mentioned earlier, Kruglov *et al.* have reported exact self-similar solutions of Eq. (4) without a source, characterized by a linear chirp and demonstrated pulse compression taking into account nonlinear soliton effects [14,18,25]. More recently, an important technology referred to as dispersion management (DM) has been developed by the researchers [2,26]. Serkin and Hasegawa have formulated the effect of varying dispersion with external harmonic oscillator potential on the soliton dynamics and have explained the concept of amplification of soliton [27]. Motivated by these works, we have analyzed solutions of Eq. (4) for pulse compression that may find application, particularly in the soliton based communication links [1] *via* asymmetric TCF. We show that it is possible to control the compression of the ψ_1 pulse in the TCF through ψ_2.

For finding solutions of Eq. (4), one writes the complex function $\psi(z,\tau)$ as

$$\psi(z,\tau) = P(z,\tau)\exp[i\Phi(z,\tau)], \quad (5)$$

where P and Φ are real functions of z and τ, where the phase has the following quadratic form:

$$\Phi(z,\tau) = a(z) + c(z)(\tau - \tau_c)^2. \quad (6)$$

Then Eq. (4) yields a self-similar form of the amplitude

$$P(z,\tau) = \frac{1}{\sqrt{1 - c_0R(z)}}Q\left(\frac{\tau-\tau_c}{1-c_0R(z)}\right)\exp\left(\frac{1}{2}S(z)\right), \quad (7)$$

where τ_c is the center of the pulse, and the functions $a(z)$, $c(z)$, $R(z)$, and $S(z)$ in the solutions given by Eqs. (6) and (7) are

$$a(z) = a_0 - \frac{\lambda}{2}\int_0^z \frac{\beta(z')dz'}{[1-c_0R(z')]^2}, \quad (8)$$

$$c(z) = \frac{c_0}{1-c_0R(z)}, \quad R(z) = 2\int_0^z \beta(z')dz', \quad (9)$$

$$S(z) = \int_0^z g(z')dz', \quad (10)$$

where a_0, λ, and c_0 are the integration constants. For the existence of the self-similar solutions, the following relationship between gain profile and distributed parameters should be maintained: $\rho(z)=\beta(z)/\gamma(z)$,

$$g(z) = \frac{1}{\rho(z)}\frac{d}{dz}\rho(z) + \frac{2c_0\beta(z)}{1-c_0R(z)}, \quad (11)$$

and the source should be of the form

$$\eta = \frac{\beta(z)}{2[1-c_0R(z)]^{3/2}}\varepsilon. \quad (12)$$

Here ε is a constant characterizing the strength of the source. In the context of TCF it should be noted that, keeping the nonlinear term for ψ_2 with appropriate distributed coefficients, one can obtain a phase Φ showing a linear chirp as required above. The spatial profile of the source can originate from the appropriate combinations of $\psi_2(z)$ and distributed Γ and α_{12}.

The function $Q(T)$ satisfies

$$Q'' - \lambda Q + 2\kappa Q^3 - \varepsilon = 0, \quad (13)$$

where the prime indicates the derivative with respect to T, where $T=(\tau-\tau_c)/[1-c_0R(z)]$ and $\kappa=-\gamma(0)/\beta(0)$.

As shown in Ref. [13], the solutions of the above equation can be obtained through a fractional transform,

$$Q(T) = \frac{A + Bf^2(T)}{1 + Df^2(T)}, \tag{14}$$

that connects the solutions of the NLSE with a source, to an elliptic equation of the type $f'' \pm af \pm \lambda f^3 = 0$. As is well known, f can be taken as any of the Jacobi elliptic functions with an appropriate modulus parameter, e.g., $\mathrm{cn}(T,m)$, $\mathrm{dn}(T,m)$, and $\mathrm{sn}(T,m)$, with amplitude and width, appropriately depending on m. Using the limiting conditions of cnoidal functions: $\mathrm{cn}^2(T,0) = \cos^2(T)$, and $\mathrm{cn}^2(T,1) = \mathrm{sech}^2(T)$; $\mathrm{dn}^2(T,0) = 1$; $\mathrm{sn}^2(T,0) = \sin^2(T)$, and $\mathrm{sn}^2(T,1) = \tanh^2(T)$, one can obtain both localized and trigonometric solutions. We list below a few interesting solutions, trigonometric, singular, and nonsingular hyperbolic ones. The singular solution indicates extreme increase in intensity due to self-focusing. Below, we give specific solutions to illustrate the compression technique. The solutions presented below are nonperturbative in the sense that they cannot be obtained through the perturbative treatment of the soliton or periodic solutions of the equations without the source. Recognizing the fact that without a source the equation describes the pulse propagation in a single-core optical fiber, it is clear that the presence of a second core significantly affects the nature of the pulses that can propagate in a twin-core fiber. We refer the interested readers to Ref. [13] for more details of the solutions.

Case (I): Trigonometric solution. For $A=0$, $\lambda=4$, and $m=0$; we find that

$$Q(T) = (\varepsilon/2)\frac{\cos^2(T)}{1 - (2/3)\cos^2(T)}, \tag{15}$$

subject to the condition on the strength of the source with the strength of the nonlinearity: $\varepsilon = \sqrt{(64/27\kappa)}$, with $\kappa > 0$.

Case (II): Hyperbolic solution. For $\kappa = -|\kappa|$, $B=0$, $\lambda=-4$, and $m=1$; we find that

$$Q(T) = (3/4)\varepsilon\frac{1}{1 - (3/2)\mathrm{sech}^2(T)}, \tag{16}$$

subject to the condition $\varepsilon = \sqrt{(64/27|\kappa|)}$. This is a singular solution. The singularity here corresponds to an extreme increase of the field amplitude due to self-focusing. For a long-haul communication network, using nanosecond pulses, the singularity of this pulse profile may correspond to the beam power exceeding the material breakdown due to self-focusing, as is known for the other nonlinear systems [28–30]. However, this catastrophic nonlinear response of the medium with the femtosecond pulses is not in conformity with the experimental observation [31].

One can also obtain pure cnoidal solutions, for different parameter values. We find that for $B=0$, one always gets singular solutions. In the case $m=1$ and $A, B \neq 0$ one can obtain exact solutions including nonsingular dark solitons. At this point it is worth mentioning that no solutions are obtained for $m=0$; $B=0$ and for $m=1$; $A=0$.

We now elucidate the compression problem of the pulse in a dispersion decreasing optical fiber. For the purpose of comparison with Ref. [14], we assume that the GVD and the nonlinearity are distributed according to the following relations:

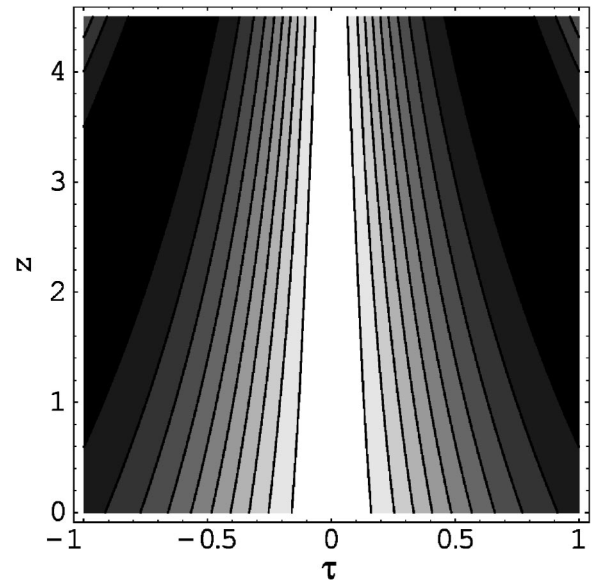

FIG. 1. Contour plot depicting the intensity of the nonlinearly compressed trigonometric solution given by Eq. (15) (in arbitrary units).

$$\beta(z) = \beta_0 \exp(-\sigma z), \quad \gamma(z) = \gamma_0 \exp(\alpha z), \tag{17}$$

where $\beta_0 \leqslant 0$, $\gamma_0 \geqslant 0$, and $\sigma \neq 0$, in which case the gain is

$$g(z) = -\alpha - \frac{\sigma(\nu - 1)}{\nu - 1 + \exp(-\sigma z)}, \tag{18}$$

where $\nu = \sigma/2c_0\beta_0$. We explicate the nonlinear compression using the trigonometric solution

$$P(z,\tau) = A(z)\frac{\cos^2[(\tau - \tau_c)/W(z)]}{1 - (2/3)\cos^2[(\tau - \tau_c)/W(z)]}, \tag{19}$$

where

$$A(z) = (\varepsilon/2)\frac{\sqrt{|\beta_0|}}{\sqrt{|\gamma_0|}}\exp\left(\frac{1}{2}(\sigma - \alpha)z\right),$$

$$W(z) = 1/\nu^{-1}[\nu - 1 + \exp(-\sigma z)].$$

We now consider an illustrative case where $\nu = 1 = \gamma_0 = |\beta_0|$ and $c_0 < 0$ so that $\sigma > 0$. We take, $\sigma = 2$ and $g(z) = -\alpha, \alpha > 0$, implying the gain is negative. The width of the solutions presented here tends to zero when $z \to \infty$.

Figure 1 shows that for the constant loss this solution can be compressed to any required degree as $z \to \infty$, while maintaining their respective original shapes, as was seen for the NLSE without source. The same can also be achieved for the dark solitons. The underlying cause of pulse compression is similar to the one in NLSE. In the presence of a linear chirp, the distributed coefficients can be absorbed in an appropriate independent variable, if the solutions are assumed to be self-similar in nature. The presence of damping term affects the amplitude of the solution without altering the basic nature of the self-similar solution.

In conclusion, we have demonstrated a different pulse compression technique based on exact solutions to the nonlinear Schrödinger-type equation interacting with a source, variable dispersion, variable Kerr nonlinearity, and variable gain or loss. A physical derivation of this system is described by including dissipation in one of the coupled equations that are appropriate for the description of pulse propagation *via* asymmetric TCF. Realizing all-optical switching processing in the present model will be of a great interest. We hope that these solutions can be launched in long-haul telecommunication networks for achieving pulse compression. We should also like to point out that, in the presence of appropriate nonlinearity, our results may find application in twin-core photonic crystal fibers [32].

K.P. wishes to thank DST, CSIR, and UGC for financial support in the form of projects.

[1] G.P. Agrawal, *Applications of Nonlinear Fiber Optics* (Academic Press, Inc., San Diego, CA, 2001).
[2] A. Hasegawa and Y. Kodama, *Solitons in Optical Communications* (Oxford University Press, Oxford, 1995).
[3] K.A. Ahmed, H.F. Liu, N. Onodera, P. Lee, R.S. Tucker, and Y. Ogawa, Electron. Lett. **29**, 57 (1993).
[4] W.J. Tomlinson, R.H. Stolen, and C.V. Shank, J. Opt. Soc. Am. B **1**, 139 (1984).
[5] L.F. Mollenauer, R.H. Stolen, J.P. Gordon, and W.J. Tomlinson, Opt. Lett. **8**, 289 (1983).
[6] E.M. Dianov, P.V. Mamyshev, A.M. Prokhorov, and S.V. Chernikov, Opt. Lett. **14**, 1008 (1989).
[7] A.M. Johnson and C.V. Shank, in *The Continuum Laser Source*, edited by R. R. Alfano (Springer-Verlag, New York, 1989).
[8] R.A. Fisher, P.L. Kelley, and T.K. Gustafson, Appl. Phys. Lett. **14**, 140 (1969).
[9] B.A. Malomed, I.M. Skinner, P.L. Chu, and G.D. Peng, Phys. Rev. E **53**, 4084 (1996).
[10] M. Liu and P. Shum, Opt. Express **11**, 116 (2003).
[11] G. Cohen, Phys. Rev. E **61**, 874 (2000).
[12] B.A. Malomed, Phys. Rev. E **51**, R864 (1995).
[13] T. Soloman Raju, C. Nagaraja Kumar, and P.K. Panigrahi, nlin.SI/0308012.
[14] V.I. Kruglov, A.C. Peacock, and J.D. Harvey, Phys. Rev. Lett. **90**, 113902 (2003).
[15] L.F. Mollenauer, R.H. Stolen, and J.P. Gordon, Phys. Rev. Lett. **45**, 1095 (1980).
[16] G.P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, Inc., San Diego, CA, 2001).
[17] A.W. Snyder and J.D. Love, *Optical Waveguide Theory* (Chapman and Hall, London, 1983).
[18] J.D. Moores, Opt. Lett. **21**, 555 (1996).
[19] S. Kumar and A. Hasegawa, Opt. Lett. **22**, 372 (1997).
[20] S.P. Brustev, A.V. Mikhailov, and V.E. Zakharov, Theor. Math. Phys. **70**, 227 (1987).
[21] M.L. Quiroga-Teixeiro, D. Anderson, P.A. Andrekson, A. Bernson, and M. Lisak, J. Opt. Soc. Am. B **13**, 687 (1996).
[22] A. Uthayakumar, K. Porsezian, and K. Nakkeeran, Jr., Pure Appl. Opt. **7**, 1459 (1998).
[23] N. Manganaro and D.F. Parker, J. Phys. A **26**, 4093 (1993).
[24] R. Driben and B.A. Malomed, Phys. Lett. A **301**, 19 (2002).
[25] T.E. Murphy, IEEE Photonics Technol. Lett. **14**, 041 (2002).
[26] M.J. Ablowitz and Z.H. Musslimani, Phys. Rev. E **67**, R025601 (2003).
[27] V.N. Serkin and A. Hasegawa, IEEE J. Sel. Top. Quantum Electron. **8**, 1 (2002).
[28] R.W. Boyd, *Nonlinear Optics* (Academic Press, Boston, 1992).
[29] M. Weinstein, Commun. Math. Phys. **87**, 567 (1983).
[30] G. Fibich and A.L. Gaeta, Opt. Lett. **25**, 335 (2000).
[31] A.L. Gaeta, Phys. Rev. Lett. **84**, 3582 (2000).
[32] W.E. Padden, M.A. van Eijekelenborg, A. Argyros, and N.A. Issa, Appl. Phys. Lett. **84**, 1689 (2004).